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On the stability of vapor film in pool film boiling

HIROAKI TANAKA

Department of Mechanical Engineering, The University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

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Abstract—The stability of vapor film in pool film boiling on a horizontal surface is considered by taking account of the effect of change in local heat transfer rate due to the perturbation of vapor film thickness. It is shown that a vapor film below a certain critical thickness becomes stable over all disturbances with any wavelength. Assuming that the average film thickness during the actual film boiling is proportional to the foregoing critical film thickness, the heat transfer coefficient can be derived. The predicted heat transfer coefficient agrees well with the recent film boiling data obtained by Shoji in a range of unprecedentedly low surface superheats using a very large surface.

1. INTRODUCTION

IN RECENT studies on boiling heat transfer, the problem of minimum heat flux point has attracted increasing interest from both practical and theoretical viewpoints. Zuber [1], using the results of Taylor Instability analysis at the liquid–vapor interface, argued that a steady supply of vapor was needed to maintain the standing wave at the interface against collapsing, and he derived the following expression for the minimum heat flux during film boiling from a horizontal surface

$$\frac{q_{\min}}{L\rho_v}\left\{\frac{\sigma g(\rho_l-\rho_v)}{(\rho_l+\rho_v)^2}\right\}^{-1/4}=\frac{\pi}{24}. \tag{1}$$

Berenson [2], assuming potential flow, inquired further into the instability problem and came to the conclusion that the effect of vapor velocity and film thickness on the liquid–vapor boundary behavior could be neglected near the minimum heat flux point, with affirmation of equation (1) but changing the right-hand side to 0.09. Then, from analysis of flow and heat transfer in the thin vapor film that existed between departing bubbles, he obtained the heat transfer coefficient near the minimum in pool film boiling from a horizontal surface as follows:

$$h=0.425\left\{\frac{\lambda_v^3 L\rho_v g(\rho_l-\rho_v)}{\mu_v\Delta T\sqrt{(\sigma/g(\rho_l-\rho_v))}}\right\}^{1/4}. \tag{2}$$

Lienhard and Wong [3] argued that the dominant unstable wavelength during film boiling above a flat plate was inapplicable to the case of boiling on a horizontal cylinder, and then developed new expressions for this case, by accounting for the effect of surface tension in the transverse direction. Dhir and Lienhard [4] made numerical evaluations of the dispersion relation for both plane and cylindrical, Taylor-unstable interfaces, considering both surface tension and fluid viscosity.

Spiegler *et al.* [5], on the other hand, proposed a new idea that the minimum of film boiling should be determined not by the hydrodynamic instability but on the basis of the maximum superheat temperature of the liquid which could be calculated satisfactorily from the Van der Waals equation. Despite the recent accumulation of experimental data of the minimum in film boiling, the controversy between the foregoing two views of its mechanism, i.e. the heat flux dominating view and the temperature dominating one, seems rather diverging than converging. A comprehensive review of the recent advances in both experimental and theoretical aspects is given by Nishio [6].

Very recently Shoji and Kaneko [7] performed an experiment on pool film boiling for water at atmospheric pressure by varying the diameter of the horizontal heating surface over a wide range between 10 and 150 mm. In the case of a large test surface, when the temperature of the surface was gradually decreased from the film boiling state at high superheat temperature difference, the heat transfer state did indeed sometimes show a kind of instability near to where the minimum used to be encountered; however, once the surface temperature passed through that region, a very stable film boiling was observed to be re-established and it was maintained down to a superheat temperature difference as small as 15 K. The results for the surface of 150 mm in diameter are reproduced from Shoji's experimental data in Fig. 1. Shoji's experiment stimulates much interest in that the minimum heat flux point was not determined uniquely for the idealistic case of an infinitely wide, horizontal surface, and further that stable film boiling was realized in a far lower heat-flux and superheat-temperature region than expected so far.

The purpose of this paper is to provide Shoji's experimental realization of film boiling in the low

NOMENCLATURE

a	amplitude of disturbance	x	horizontal axis fixed on the heating surface
g	acceleration of gravity	y	vertical distance from the heating surface.
h	heat transfer coefficient		
k	wave number of disturbance, $2\pi/\lambda$		
k_c	critical wave number		Greek symbols
L	latent heat of vaporization	δ	vapor film thickness
p_l	pressure on the liquid side of the liquid–vapor interface	δ_0	mean vapor film thickness
p_v	pressure in the vapor film	δ_c	uppermost, stable film thickness
q	heat flux	δ_{cmin}	least uppermost, stable film thickness
q_{min}	minimum film boiling heat flux	κ_v	thermal diffusivity of vapor
s	growth coefficient of disturbance	λ	wavelength
t	time	λ_c	critical wavelength
ΔT	temperature difference between heating surface and saturated liquid	λ_v	thermal conductivity of vapor
U_m, U'_m	mean vapor flow velocity induced by disturbance	μ_v	viscosity of vapor
		ρ_l	density of liquid
		ρ_v	density of vapor
		σ	surface tension.

superheat temperature region with theoretical justification. In doing so, a new aspect of the mechanism of film boiling heat transfer is discovered, which is expected to contribute to correct understanding of the phenomena of the minimum of film boiling. The specific feature of the present theory is that the effect of change in local heat transfer rate due to small disturbances of vapor film thickness is taken into account. This effect proves to stabilize the vapor film.

2. THEORY

Stationary film boiling is assumed to be taking place above an infinitely wide, horizontal surface of a constant temperature (Fig. 2). The distance y is measured upward from the surface, and the x-axis is fixed on the surface. A two-dimensional state in the x–y plane is assumed. Keeping in mind the very calm film boiling with rare departure of bubbles that was observed by

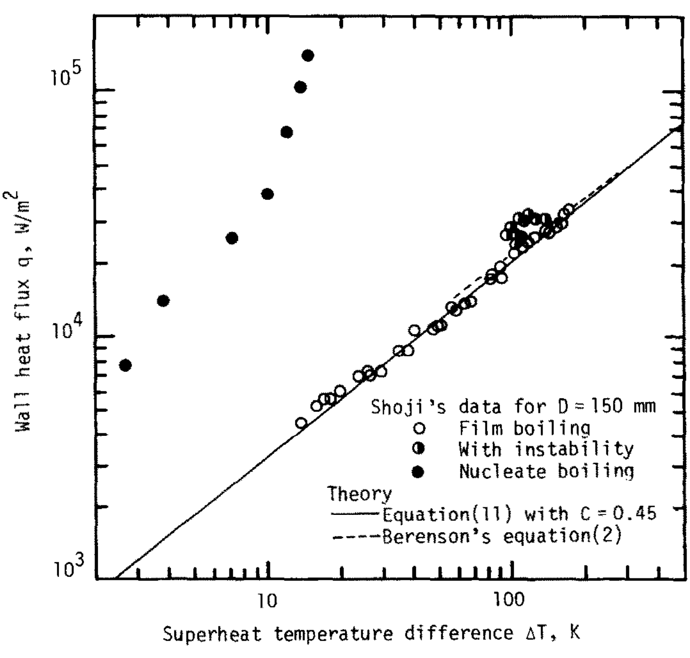

FIG. 1. Shoji's film boiling data [7] in a very low superheat temperature region, compared with the present theory.

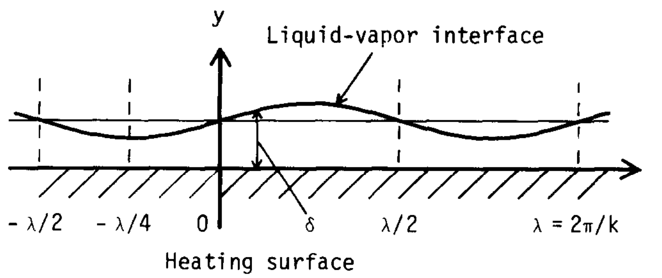

FIG. 2. Two-dimensional model for film boiling on a horizontal surface.

Shoji and Kaneko [7] in the low superheat temperature region, and assuming an appropriate subcooling of the liquid, let us suppose that thermal as well as dynamic equilibrium holds when the vapor film has a uniform thickness of δ_0.

The interface is assumed to be disturbed by waves in the form

$$\delta = \delta_0 + a \sin kx \cdot e^{st} = \delta_0 + a \sin 2\pi \frac{x}{\lambda} \cdot e^{st}. \quad (3)$$

Here, the amplitude a of the disturbance is assumed to be so small that terms higher than the second power of a/δ_0 can be neglected in the following analysis. Moreover, we assume the condition $\delta_0 \ll \lambda$, which will be well justified for an actual case of water afterwards.

This disturbance induces flow from the region where the film thickness is decreasing toward where it is increasing. The mean velocity of the induced flow is obtained by the time rate of change of the integration of the film volume from $x = -\lambda/4$, where film deformation is symmetric, to $x = x$:

$$\begin{aligned} U_m &= -\frac{1}{\delta}\frac{\partial}{\partial t}\int_{-\lambda/4}^{x} \delta \, dx \\ &= -\frac{1}{\delta}\int_{-\pi/2k}^{x} as \sin kx \cdot e^{st} \, dx \\ &= \frac{a}{\delta}\frac{s}{k}\cos kx \cdot e^{st} \\ &= \left\{\frac{a}{\delta_0} + O\left(\frac{a}{\delta_0}\right)^2\right\}\frac{s}{k}\cos kx \cdot e^{st}. \quad (4) \end{aligned}$$

In addition to this, the deviation of the film thickness from δ_0 invalidates the thermal equilibrium, resulting in evaporation or condensation depending on whether the film thickness is smaller or larger than δ_0. Here, we can assume linear temperature distribution in the vapor film, since the time constant for thermal conduction in the film, δ_0^2/κ_v, turns out to be several orders of magnitude smaller than that of the disturbance, $1/|s|$, as will become clear afterwards. Therefore, the deviation Δq from the equilibrium heat flux q is given by

$$\begin{aligned} \frac{\Delta q}{q} &= \frac{\frac{1}{\delta} - \frac{1}{\delta_0}}{\frac{1}{\delta_0}} \\ &= \frac{\delta_0 - \delta}{\delta} = -\frac{a}{\delta}\sin kx \cdot e^{st}. \end{aligned}$$

The corresponding volume rate of evaporation (condensation) per unit time and unit area is

$$\begin{aligned} \frac{\Delta q}{L\rho_v} &= -\frac{q}{L\rho_v}\frac{a}{\delta}\sin kx \cdot e^{st} \\ &= -\frac{q}{L\rho_v}\left\{\frac{a}{\delta_0} + O\left(\frac{a}{\delta_0}\right)^2\right\}\sin kx \cdot e^{st}. \end{aligned}$$

Integrating this from $x = -\lambda/4$ to x gives the following mean velocity of the flow induced in the film by the thermal inequilibrium:

$$\begin{aligned} U_m' &= -\frac{1}{\delta}\int_{-\lambda/4}^{x} \frac{q}{L\rho_v}\frac{a}{\delta_0}\sin kx \cdot e^{st} \, dx \\ &= \frac{q}{L\rho_v}\frac{a}{\delta_0^2 k}\cos kx \cdot e^{st}. \quad (5) \end{aligned}$$

The flow in the film may be approximated by the plane Poiseuille flow:

$$u = 6(U_m + U_m')\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right).$$

Then, the pressure gradient in the vapor film becomes

$$\frac{\partial p_v}{\partial x} = -12\mu_v \frac{U_m + U_m'}{\delta^2}.$$

Substituting U_m and U_m' from equations (4) and (5) into the above equation yields

$$\frac{\partial p_v}{\partial x} = -12\mu_v \frac{a}{\delta_0^3 k}\left(s + \frac{q}{L\rho_v \delta_0}\right)\cos kx \cdot e^{st}.$$

Thus we arrive at the following expression for the pressure distribution in the vapor film, where the pressure at $x = 0$ is taken as a standard

$$p_v = \int_0^x \frac{\partial p_v}{\partial x}\,dx = -12\mu_v \frac{a}{\delta_0^3 k^2} \times \left(s + \frac{q}{L\rho_v\delta_0}\right)\sin kx \cdot e^{st}. \quad (6)$$

The pressure on the liquid side of the liquid–vapor interface is, by neglecting the kinetic energy of the liquid and taking only the contribution from gravity into account

$$p_l = -\rho_l g a \sin kx \cdot e^{st}. \quad (7)$$

Here, the pressure zero is intentionally taken at $x = 0$, where the film thickness is kept constant at its mean value δ_0 and further the curvature of the liquid–vapor interface vanishes. The balance of pressures across the liquid–vapor interface taking account of the effect of surface tension is expressed by

$$p_l - p_v = \sigma \frac{\partial^2 \delta}{\partial x^2}.$$

Substituting equations (3), (6) and (7)

$$-\rho_l g a \sin kx \cdot e^{st} + 12\mu_v \frac{a}{\delta_0^3 k^2}\left(s + \frac{q}{L\rho_v\delta_0}\right) \times \sin kx \cdot e^{st} = -\sigma a k^2 \sin kx \cdot e^{st}.$$

This reduces to

$$s + \frac{q}{L\rho_v\delta_0} = \frac{(\delta_0 k)^3}{12\mu_v}\left(\frac{\rho_l g}{k} - \sigma k\right). \quad (8)$$

Equation (8) can be interpreted as follows. First, in the case $q \to 0$, or, supposing a noncondensable gas film δ_0 thick rather than a vapor film, a disturbance will grow or damp depending on whether its wave number k is smaller or larger than the well-known, critical wave number $k_c = \sqrt{(\rho_l g/\sigma)}$, since the right-hand side of equation (8), accordingly the growth coefficient s, changes its sign at k_c. (For water at 100°C, $k_c = 399\,\mathrm{m^{-1}}$ and the corresponding wavelength $\lambda_c = 15.7$ mm.) For a disturbance with a wavelength smaller than λ_c, the surface tension surpasses the gravity effect and makes the liquid–vapor interface return flat. For a disturbance with a longer wavelength than λ_c, on the other hand, the gravity effect predominates and amplifies the disturbance. Whether a disturbance grows or damps, the time constant of its change, $1/|s|$, is determined by the agency of viscosity and it is inversely proportional to the third power of the film thickness, thus increasing very sharply with decrease in the film thickness. For instance, supposing water at 100°C and taking $k = k_c/\sqrt{2}$ the time constant $1/s$ amounts to 3.15 s for $\delta_0 = 0.05$ mm, while it is 0.394 s for $\delta_0 = 0.1$ mm.

When the second term on the left-hand side of equation (8) is considered, s is shifted in the minus direction that much, even if the right-hand side has a plus sign. When the heat transfer effect is involved, the evaporation and condensation caused by the disturbance acts in a way to stabilize the vapor film. Furthermore, a thin enough vapor film is always stable, because with decreasing δ_0 the right-hand side of equation (8) decreases in proportion to the third power of δ_0 while the second term on the left-hand side increases in inverse proportion to δ_0. Letting $s = 0$ in equation (8) and solving for δ_0 gives the uppermost, stable film thickness for a disturbance with wave number k:

$$\delta_c = \left\{\frac{1}{k^3}\frac{q}{L\rho_v}\frac{12\mu_v}{(\rho_l g/k) - \sigma k}\right\}^{1/4}. \quad (9)$$

Putting $k = k_c/n = (1/n)\sqrt{(\rho_l g/\sigma)}$ yields

$$\delta_c = \left\{n^3\left(\frac{\sigma}{\rho_l g}\right)^{3/2}\frac{q}{L\rho_v}\cdot\frac{1}{n-(1/n)}\frac{12\mu_v}{(\sigma\rho_l g)^{1/2}}\right\}^{1/4} = 12^{1/4}\frac{n}{(n^2-1)^{1/4}}\left\{\frac{q}{L\rho_v}\frac{\sigma\mu_v}{(\rho_l g)^2}\right\}^{1/4}.$$

Therefore, the least uppermost, stable film thickness occurs at $n = \sqrt{2}$, and it amounts to

$$\delta_{c\min} = 2\times 3^{1/4}\left\{\frac{q}{L\rho_v}\frac{\sigma\mu_v}{(\rho_l g)^2}\right\}^{1/4} = 2.63\left\{\frac{q}{L\rho_v}\frac{\sigma\mu_v}{(\rho_l g)^2}\right\}^{1/4}. \quad (10)$$

For instance, equation (10) gives $\delta_{c\min} = 0.232$ mm for $q = 10^4\,\mathrm{W\,m^{-2}}$. From Shoji's experimental data in Fig. 1, on the other hand, the mean film thickness is estimated to be 0.1 mm for the same heat flux.

At this juncture, let us suppose that in actual film boiling the effective vapor film thickness averaged over the period of bubble departure is C times $\delta_{c\min}$. Then, the heat transfer coefficient is expressed as $h = \lambda_v/(C\delta_{c\min})$. By introducing equation (10), this can be rewritten as

$$\frac{h}{\lambda_v}\left\{\frac{q}{L\rho_v}\frac{\sigma\mu_v}{(\rho_l g)^2}\right\}^{1/4} = \frac{1}{2\times 3^{1/4}C}. \quad (11)$$

This equation implies $q \propto \Delta T^{4/5}$, which tendency is clearly demonstrated by Shoji's data in Fig. 1. Furthermore, putting $C = 0.45$ results in the numerical value of 0.85 for the right-hand side of equation (11). The heat transfer relation predicted by assuming this numerical constant is plotted by a solid line in Fig. 1. The prediction is quantitatively in good agreement with the experimental data. The numerical constant C may be determined in relation to the thickness of the film that will be left immediately after the bubble departure, but $C = 0.45$ seems to be in a reasonable range.

In passing, Berenson's equation (2) is rewritten in the following form similar to equation (11):

$$\frac{h}{\lambda_v}\left\{\frac{1}{L\rho_v}\frac{\lambda_v \Delta T}{\sqrt{(\sigma/g(\rho_l-\rho_v))}}\frac{\sigma\mu_v}{g^2(\rho_l-\rho_v)^2}\right\}^{1/4} = 0.425. \quad (12)$$

Thus, the critical wavelength divided by 2π appears as a length parameter in the corresponding heat flux factor. Despite this difference, equations (11) and (12) show fair agreement around the minimum heat flux point observed previously, as seen in Fig. 1.

3. CONCLUDING REMARKS

Although the vapor film in film boiling has always been considered to be essentially unstable because of Taylor Instability, it proves to have a trend to stability when the heat transfer effect is taken into account. As a result, a vapor film below δ_{cmin} in thickness becomes stable for all disturbances with any wave number. Shoji's film boiling data in the low superheat region obtained for the test surface of 150 mm in diameter can be well explained by assuming that the effective film thickness averaged over the period of bubble departure is about half of δ_{cmin}.

A new view of the phenomena of the minimum of film boiling is expected to result from the present viewpoint that the vapor film is stable in its nature. In developing the new view, the following facts are noteworthy. Shoji and Kaneko [7] experienced some cases where the film boiling state transferred from a high superheat region to the low superheat region very smoothly without showing any singularity near where the minimum used to be encountered. Corresponding to this fact, equation (11), in agreement with equation (12) in the minimum heat flux region, has possibility of its application being extended to the higher superheat range.

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SUR LA STABILITE D'UN FILM DE VAPEUR DANS L'EBULLITION EN RESERVOIR

Résumé—La stabilité du film de vapeur sur une surface horizontale dans l'ébullition en réservoir, est considérée en prenant en compte l'effet du changement de transfert local de chaleur dû à la perturbation de l'épaisseur du film de vapeur. On montre qu'un film de vapeur, au dessous d'une certaine épaisseur critique, est stable pour toute perturbation de longueur d'onde quelconque. On peut calculer le coefficient de transfert en supposant que l'épaisseur moyenne du film pendant l'ébullition est proportionnelle à l'épaisseur critique. Le coefficient ainsi déterminé s'accorde bien avec les données expérimentales obtenues récemment dans un domaine de surchauffe faible pour des surfaces très étendues.

ZUR STABILITÄT DES DAMPFFILMES BEIM BEHÄLTERSIEDEN

Zusammenfassung—Die Stabilität des Dampffilmes beim Behälterfilmsieden an einer horizontalen Oberfläche wird durch Berücksichtigung des Einflusses des unterschiedlichen lokalen Wärmeübergangs aufgrund der Veränderung der Dampffilmdicke betrachtet. Es wird gezeigt, daß ein Dampffilm unterhalb einer gewissen kritischen Dicke gegenüber beliebigen Störungen stabil ist. Unter der Annahme, daß die mittlere Filmdicke während des Filmsiedens proportional zu dieser kritischen Filmdicke ist, kann der Wärmeübergangskoeffizient berechnet werden. Die berechneten Wärmeübergangskoeffizienten stimmen gut mit neueren Messungen beim Filmsieden überein, welche von Shoji in einem Bereich von bisher noch nicht untersuchten kleinen Wandüberhitzungen an einer sehr großen Oberfläche durchgeführt wurden.

ОБ УСТОЙЧИВОСТИ ПЛЕНКИ ПАРА ПРИ ПЛЕНОЧНОМ КИПЕНИИ В БОЛЬШОМ ОБЪЕМЕ

Аннотация—Рассматривается устойчивость пленки пара при пленочном кипении в большом объеме на горизонтальной поверхности с учетом изменения интенсивности локального теплообмена из-за возникающих в пленке возмущений. Показано, что ниже определенной критической толщины пленка пара становится устойчивой при возмущениях любой длины волны. Предполагая, что осредненная толщина пленки в реальных условиях пленочного кипения пропорциональна вышеуказанной ее критической толщине, можно определить коэффициент теплообмена. Рассчитанный коэффициент теплообмена хорошо согласуется с данными Шойджи по пленочному кипению в диапазоне крайне малых перегревов, которые были получены для поверхности с очень большой площадью.